Dynamic modelling of fuel accumulation

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Abstract

Post-fire vegetation recovery is an important consideration in bushfire risk management, both in terms of assessing future fire risk as well as balancing the ecological requirements associated with a vegetation stand. Traditionally, the accumulation of fuel after a fire has been estimated using the Olson model, which is determined as a simple balance between the rates at which fuels accumulate and the rate at which they decay.

Over the years, and especially more recently, a number of authors have highlighted some limitations of the Olson model. In particular, it has been shown that the Olson model does not necessarily account for the development of an understorey shrub layer, such as often occurs in montane forest environments. In some situations, this shrub layer can produce a very rapid increase in fuels after a fire, but will eventually senesce as the forest as a whole matures. The ecological and fire risk implications of this are significant; while a more mature forest represents the preferred ecological state and may be less flammable, to reach maturity the forest must progress through a stage of very high flammability. Balancing the risk of catastrophic fires with desirable ecological outcomes is an ongoing challenge for land managers.

In this paper we present a mathematical framework for modelling fuel accumulation based on dynamical systems theory. The model permits the interaction of two different vegetation species (trees and shrubs), each of which is subject to certain environmental carrying capacities, and accounts for their combined effect on fuel accumulation. In particular, the framework allows for the effects of senescence of the shrub layer, before a steady state is gradually attained. Possibilities for extending dynamic modelling framework to simulate more complex forest ecosystems, and its implications for fire risk and fire ecological management, are also discussed.

Keywords: Olson model, mathematical modelling, vegetation growth, fuel accumulations, senescence phase, bushfire

1. Introduction

Forest stands can be thought of as having a specific structure defined by the distribution layers of trees, shrubs and surface cover, the latter of which is comprised of fine dead litter and woody materials. In fire-prone ecosystems, the amount of surface fuel can be taken as a measure of the fire hazard, which is then managed through prescribed burning programs. After a fire, which removes a portion of the surface fuel, litter and woody debris will gradually re-accumulate. To assess the manner in which fuels accumulate over time in a particular forest stand, fire managers rely on fuel accumulation models to account for the competing processes of litter production and decomposition. For instance, these models are used to help predict when fuel loads will again reach hazardous levels, so that future hazard reduction burning can be appropriately scheduled.

Understanding how vegetation responds post-fire is therefore important, both in terms of assessing future fire risk as well as balancing the ecological requirements associated with a vegetation stand. Indeed, in some instances prescribed burning that is performed too frequently, or not frequently enough, has produced unforeseen effects that do not meet with overall management objectives (Fernandes and Botelho 2003). For example, this can be due to the build up of species with very high litter production rates, which then results in highly flammable environments. A number of authors have reported on such occurrences, where recent burning produces more flammable forests compared to mature forests that are long unburnt (Dixon et al. 2018; Zylstra 2018).

The most commonly used approach to modelling fuel accumulation was developed by Olson (1963). In this approach the function $x(t)$ is used to denote the amount of fuel, and is determined by the following equation:

$$\frac{dx}{dt} = L - kx, \qquad x(0) = 0. \tag{1}$$

The initial condition $x(0) = 0$ is taken as such because it is assumed that no litter exists immediately after the fire. The parameter L describes the rate of litter deposition, while the parameter k describes the rate at which litter decomposes. Solution of the differential equation (1) produces the function

$$x(t) = x_{ss}\left(1 - e^{-kt}\right), \tag{2}$$

with $x_{ss} = L/k$ and $x_0 = 0$. This function is commonly referred to as the 'negative-exponential model' or the 'Olson curve', and has been widely accepted in the fire-ecology literature. Researchers typically use the function in eq. (2) to estimate the time for the forest to reach a steady state density, x_{ss}.

A number of authors have modified the Olson model to take into account the possibility that not all of the fuel is consumed by the fire (Birk and Simpson 1980; Raison et al. 1983). This amounts to changing the initial condition to $x(0) = x_0 \geq 0$. The corresponding solution in this case is:

$$x(t) = x_{ss}\left(1 - e^{-kt}\right) + x_0\, e^{-kt}. \tag{3}$$

Fensham (1992) claimed that eq. (3) with $x_0 \geq 0$ provided a better fit to data compared to the original model (2). Tolhurst and Kelly (2003) also fit their data to a model of the form of eq. (3) using regression analysis to compensate for the non-zero condition of post-fire litter. This study was applied to different litter components to account for effects produced by canopy structure and forest type.

Over the years, a number of studies have also indicated that assuming a constant rate of litter decomposition k may not be valid. Indeed, it has been postulated that as the litter bed reestablishes itself after a fire, the litter substrate should be viewed as being composed of two different components (x_1 and x_2), which decay at different rates, k_1 and k_2, respectively (Bunnell and Tait 1974; Hunt 1977; O'Connell 1987; Ezcurra and Becerra 1987; Bresnehan 2003). To account for this possibility, the so-called 'double-exponential' model was introduced, and is expressed as

$$x(t) = x_1 e^{-k_1 t} + (x_0 - x_1)e^{-k_2 t}, \tag{4}$$

where $x(0) = x_0 = x_1 + x_2$.

Other authors have presented models that account for temporal variation in both the decay rate k and the litter production rate L. For example, Mercer et al. (1995) adapted eq. (1), allowing $L(t)$ and $k(t)$ to vary sinusoidally with time, in order to account for seasonal differences in litter production and decay. A similar sinusoidal model for seasonal litter fall was also used by Cook et al. (2017).

Walker (1981) considered that litter production should actually depend on the amount of foliage in the forest canopy. To account for this, he related litter production to foliage projective cover (FPC), which provides a measure of the percentage of ground cover occupied by vertical forest stand. Walker (1981) derived a relationship of the form:

$$L = a + b\mathtt{FPC} + c\mathtt{FPC}^2. \tag{5}$$

While eq. (5) stipulates how litter production changes with foliage cover, it does not directly allow for changes in L as a forest regrows after a fire. Moreover, Specht and Morgan (1981) assumed a relationship between foliage height and ground cover and then confirmed it by analysing data using regression analysis. It would therefore seem reasonable to include a process that accounts for the growth of trees and allows for the effects of understorey cover, including the possibility of senescence, in modelling fuel accumulation as a forest matures.

In the next section we present a mathematical model that simulates the post-fire regrowth of two main vegetation species in a forest, and link it to surface fuel accumulation. The model assumes logistic growth for the two species, and incorporates species competition (e.g. through the effects of shading) evaluated from stand structure of vegetation growth are developed as components in ordinary differential equations. This observation is important in managing and protecting a forest long-term.

2. A model incorporating regrowth and species competition

Vegetation regrowth is determined through a complex, symbiotic relationship between new re-sprouting species, which interact through competition for nutrients and light. As a starting point to incorporating these complex dynamics, we consider two species (trees and shrubs) and propose using logistic growth models to model changes in the vegetation density of the two species. To account for competition between the two species, we include a Holling Type I response function (Collings 1997). A schematic representation of the model is shown in Fig. 1.

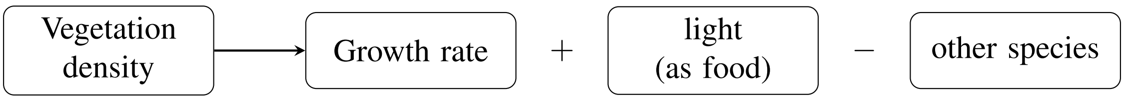

Fig. 1: An equivalent illustration for one species vegetation density with growth rate as defined in competition of getting lights and other species

We therefore consider the following system of ordinary differential equations, in which M represents the density of mature trees, S represents the shrub density, and X represents the accumulated density of surface fuel.

$$\frac{dM}{dt} = r_1 M\left(1 - \frac{M}{K_1}\right), \quad M(0) = M_0 \geq 0, \tag{6}$$

$$\frac{dS}{dt} = r_2 S\left(1 - \frac{S}{K_2}\right) - aMS, \quad S(0) = S_0 \geq 0, \tag{7}$$

$$\frac{dX}{dt} = f(M,S) - k\,X, \quad X(0) = 0. \tag{8}$$

Here the parameter r_1 is an intrinsic growth rate of mature trees, and K_1 is the carrying capacity of mature trees. The parameter r_2 is the intrinsic growth rate of shrubs, and K_2 the carrying capacity of shrubs. The parameter a is the competition factor, which defines the strength of the interaction term aMS. This term represents the limiting effect of shading on shrub growth as the tree density increases. We assume that the function $f(M,S)$ in eq. (8) is given by

$$f(M,S) = \alpha M + \beta S,$$

where α and β are the rates of litter production by mature trees and shrubs, respectively. Note that setting $f(M,S) = L$ recovers the Olson model, eq. (1).

The analysis is simplified by introducing the dimensionless variables

$$M^* = \frac{M}{K_1}, \quad S^* = \frac{S}{K_2}, \quad X^* = kX \quad \text{and} \quad \tau = r_2 t$$

which yields the non-dimensionalised system

$$\frac{dM^*}{d\tau} = \lambda_1 M^* (1 - M^*), \tag{9}$$

$$\frac{dS^*}{d\tau} = S^*(1 - S^*) - \lambda_2 M^* S^*, \tag{10}$$

$$\frac{dX^*}{d\tau} = \lambda_3 M^* + \lambda_4 S^* - \lambda_5 X^*, \tag{11}$$

where the parameters

$$\lambda_1 = \frac{r_1}{r_2}, \quad \lambda_2 = \frac{aK_1}{r_2}, \quad \lambda_3 = \frac{\alpha k K_1}{r_2}, \quad \lambda_4 = \frac{\beta k K_2}{r_2}, \quad \lambda_5 = \frac{k}{r_2}.$$

An example solution of the system (9, 10, 11) is shown in Fig. 2. The parameter values assumed here are

$$\lambda_1 = 0.3; \quad \lambda_2 = 0.75; \quad \lambda_3 = 0.12; \quad \lambda_4 = 0.4; \quad \lambda_5 = 0.56$$

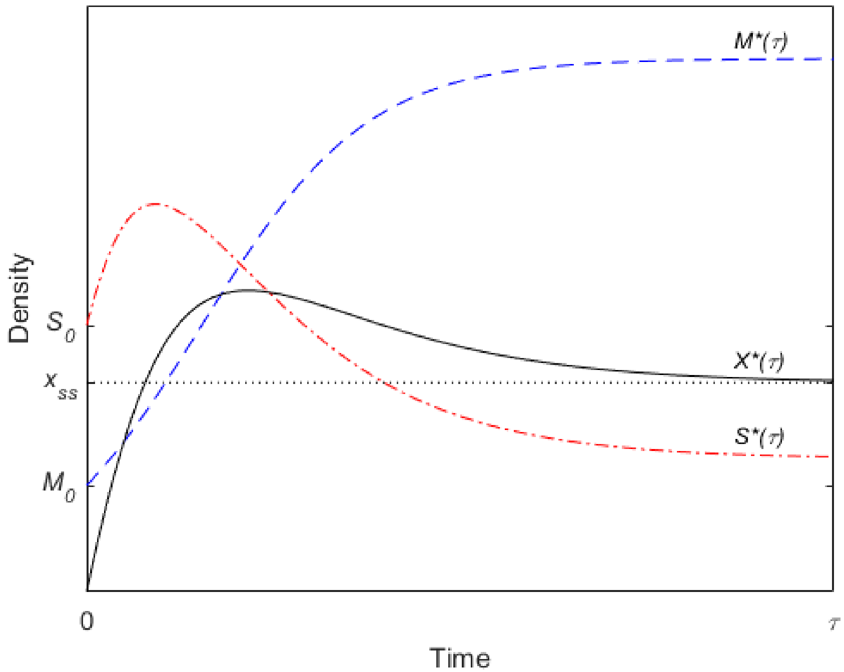

Fig. 2: Fuel accumulation over time arising from the interaction of tree and shrub species. The blue dashed curve represents mature tree density, with $M_0 = 0.2$, the red dashed curve is the shrub density, when $S_0 = 0.5$, and the black curve is the litter density, which accumulates from an initial density of $X_0 = 0$.

Fig. 2 illustrates the effects of vegetation regrowth and of species interaction on fuel accumulation. The parameter values are such that shrub growth is dominant immediately post-fire. Hence, mature trees start with a lower density than shrubs, and the rate of increase of shrub density is initially much higher than the rate of increase of trees. The increase in tree density M follows a logistic growth function, and asymptotically approaches the carrying capacity K_1. Note that the tree growth curve possesses an inflection point, which occurs near the point of maximum shrub density.

Beyond this point the density of shrubs start to decline as competition (e.g. shading) from the growing number of mature trees starts to dominate the dynamics and causes the shrubs to senesce towards a lower equilibrium density. The dynamic interaction between the two vegetation species initially causes the litter density to rise rapidly, then after the shrubs start to senesce the litter density reaches a maximum before falling to an equilibrium level x_{ss}. The implication is that, under this scenario, the peak fire hazard of the long unburnt forest is less than a forest that has been burnt more recently. This is similar to the patterns reported by Dixon et al. (2018) and Zylstra (2018).

3. Conclusion

A fuel accumulation model that accounts for vegetation regrowth and species interaction has been presented. The model can be seen as a generalisation of the Olson model, but one which is able to emulate patterns of fuel accumulation similar to those reported in recent literature. In particular, the model allows for the possibility that long unburnt forests are less flammable that more recently burnt ones – this is manifestly at odds with the behaviour of the Olson model, which dictates that fuel loads continually increase to a steady state level.

Although still in an initial phase of development, the modelling framework presented here has considerable promise, as it can be calibrated on real fuel accumulation data to provide more faithful estimates of the way fuels accumulate. Moreover, the modelling framework could easily be extended further to account for interactions between more than two species and for more complex litter decomposition dynamics.

The model has a number of significant implications for fuel management. In particular, it offers a more comprehensive theoretical basis for making decisions aimed at optimising the balance between hazard reduction and ecological objectives. For example, decisions based on the Olson model alone, will favour burning before the forest matures, since it implies ever increasing levels of fuel. On the other hand, the model presented here, and as illustrated in Fig. 2, allows for a more nuanced approach to bushfire risk management.

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